

## NARROW BANDSTOP FILTERS

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**Abstract**--The synthesis of narrow bandstop filters with arbitrary stopband and equiripple passband responses is demonstrated. A new transformed frequency variable is used for iterative approximation with automatic bandwidth adjustment and prototype circuit realization.

## INTRODUCTION

A design method is presented using a new transformed frequency variable for narrow bandstop filters. With this method, the most general class of equiripple passband, narrow bandstop filters can be designed to meet arbitrary stopband requirements with symmetric or asymmetric frequency responses.

Symmetric frequency responses include conventional Chebyshev and elliptic function filters, and any other frequency-symmetric distribution of the stopband loss pole frequencies.

Asymmetric frequency responses include any other arbitrary distribution of the stopband loss pole frequencies. A simple example of an asymmetric response is shown in Figures 1 and 2, which are the stopband and SWR responses of a five-resonator narrow bandstop filter. This filter has an asymmetric stopband with two bandstop resonators tuned to a lower frequency and three resonators tuned to a higher frequency. The resulting equiripple SWR response has two reflection minima in the lower passband, and three minima in the upper passband.

Both the approximation and realization steps of synthesis are performed in the new transformed variable, the use of which simplifies the iterative approximation of equiripple passband, general stopband responses, and, with care, achieves high numerical accuracy in the realization of the prototype element values [1]. Auxiliary transformed variables are included for automatic bandwidth adjustment in the approximation procedure [2]. The general steps taken in the narrow bandstop synthesis are similar to those used for the narrow bandpass filter [3].

## FILTER SPECIFICATIONS

The narrow-bandstop filter must pass signals below and above a pair of specified passband edge frequencies ( $p_1, p_2$ ) with a specified maximum reflection, and it must reject signals between a pair of stopband edge frequencies ( $s_1 > p_1, s_2 < p_2$ ) by specified values of minimum loss. The filter designer will obtain these values from the imposed requirements for the filter, modifying them to include margins for practical tolerances, tuning capability, environmental conditions, and any other "rule-of-thumb" based on experience.

Because there is no restriction on the symmetry of the frequency response, the definition of a "center" frequency ( $f_0$ ) becomes somewhat arbitrary. A very useful choice of  $f_0$  is

$$f_0 = (p_2 \cdot s_1 - p_1 \cdot s_2) / [(p_2 - p_1) - (s_2 - s_1)],$$

which divides the stopband width ( $s_2 - s_1$ ) and specified passband width ( $p_2 - p_1$ ) into similar proportions.

For a specific filter design, the actual equiripple passband edge frequencies ( $f_1, f_2$ ) will be determined during the approximation step. A satisfactory design will meet the conditions  $f_1 \geq p_1$  and  $f_2 \leq p_2$ . The equiripple passband width ( $f_2 - f_1$ ) is referred to as the filter band-width.

## TRANSFORMED VARIABLES

A transformed variable ( $z$ ) will map the passbands of the bandstop filter ( $f \leq f_1, f \geq f_2$ ) onto the entire imaginary axis in the  $z$  plane, and the stopband ( $s_1 \leq f \leq s_2$ ) to the positive real axis. The new transformation

$$z^2 = (f - f_1) / (f_2 - f), \quad \text{Re}(z) \geq 0$$

accomplishes the desired mapping. Approx-

priate auxiliary transformed variables for automatic bandwidth adjustment are

$$x = (f-s_1)/(s_2-f)$$

$$y = (f_2-f_1)/(s_2-s_1)-1$$

where  $x$  and  $y$  are the stopband and bandwidth transformed variables, respectively. Stopband loss poles will map onto  $x \geq 0$ , independently of the filter bandwidth, and the filter bandwidth, which is always greater than the stopband width, will map onto  $y > 0$ .

A convenient way to relate  $f_1$  and  $f_2$  to the filter bandwidth is to require that  $f_0$  will also divide that bandwidth into the same proportions as the other bandwidths. Then the transform of  $f_0$  becomes a constant, independent of the bandwidth:

$$z_0^2 = z^2(f_0) = (s_1-p_1)/(p_2-s_1).$$

As a result, the actual passband edge frequencies ( $f_1, f_2$ ) will vary so that in the limit that the filter bandwidth is equal to the stopband width ( $y = 0$ ), they will coincide with the stopband edge frequencies ( $s_1, s_2$ ), and when the bandwidth is equal to the specified passband width, they will coincide with the specified passband edge frequencies ( $p_1, p_2$ ).

By analogy to a narrow bandpass filter whose prototype is a normalized lowpass filter, the prototype for the narrow bandstop filter is a normalized highpass filter [4]. In the normalized frequency domain ( $\omega$ ) for the prototype the transformed variable is

$$z^2 = (1+\omega)/(1-\omega), \text{ Re}(z) \geq 0,$$

where  $f_1$  maps to  $\omega = -1$ ,  $f_2$  maps to  $\omega = +1$ , and the stopband frequencies map into  $-1 < \omega < +1$ .

#### DESIGN EXAMPLE

The five-resonator filter described above will be used to illustrate the new method. The filter specifications are: passband from  $p_1 = 950$  MHz to  $p_2 = 1050$  MHz and 1.1 maximum SWR; stopband with 45 dB minimum from  $s_1 = 1000$  to  $1010$  MHz and 60 dB minimum from  $1015$  to  $s_2 = 1020$  MHz.

A Chebyshev rational function [3] was used for approximation of an equiripple passband filter. The loss response of the filter was optimized by adjusting the two loss pole frequencies and the filter bandwidth, so that the stopband loss exactly met the specifications. Figures 1 and 2 are the resulting stopband and equiripple SWR responses, where  $f_0 = 1012.50$  MHz,  $f_1$

$= 981.47$  MHz,  $f_2 = 1031.11$  MHz, the loss pole frequencies are 1002.48 and 1017.95 MHz, and the frequency of the minimum in the stopband response is 1009.09 MHz.

After the approximation step, the transducer and characteristic functions were generated, leading to the two-port open-circuit immittance parameters. The normalized highpass prototype was realized, with shunt bandstop resonators, separated by constant (frequency independent) phase shifters which were realized directly (rather than indirectly from admittance inverters and shunt susceptances [4]). A symmetric circuit, shown in Figure 3, resulted from the removal of a lower frequency resonator from each end, with the three higher frequency resonators in the middle. The center resonator is shunted by a constant susceptance, which accounts for the mismatch in the response at infinite frequency. All calculations were performed with polynomials in  $z$ .

Conversion of the highpass prototype into a normalized transmission line prototype is shown in Figure 4, with stopband and SWR responses shown in Figures 5 and 6. Depending on the filter structure chosen, this prototype must be converted again into a practical bandstop circuit. Although the stopband response is very close to that of the original approximation, further optimization (including tuning) will restore the SWR response.

#### REFERENCES

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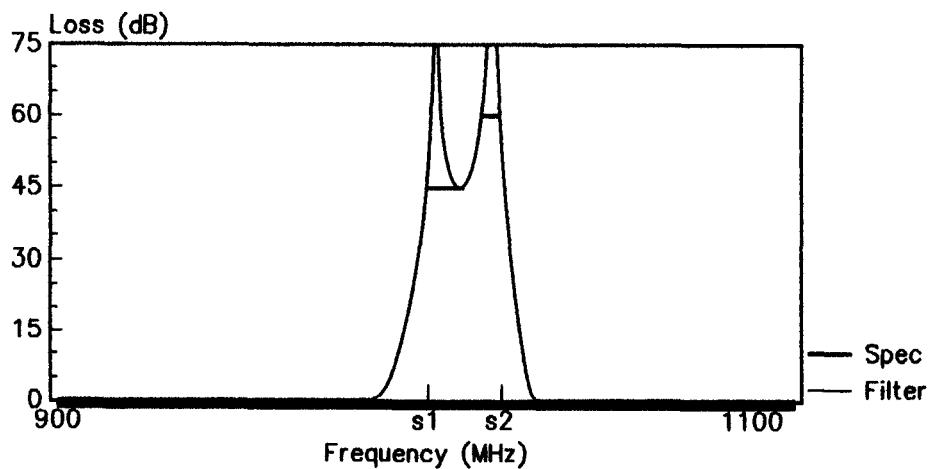


Fig. 1. Loss response, Chebyshev rational function.

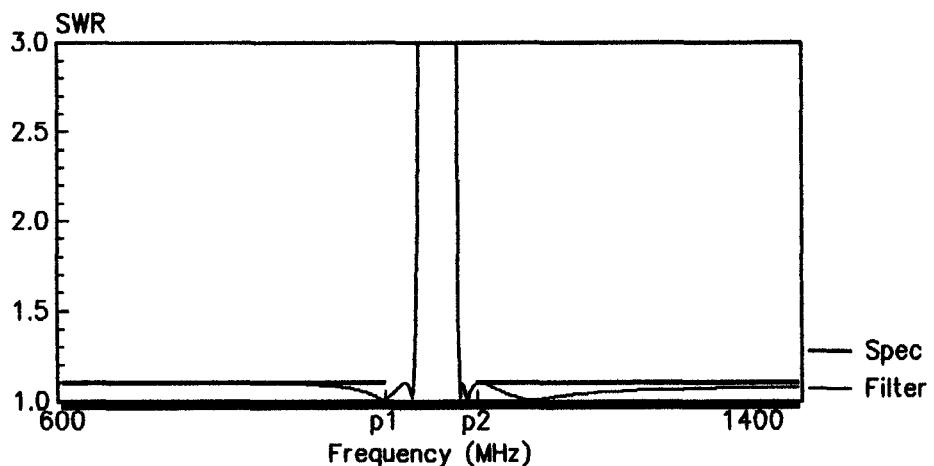


Fig. 2. SWR response, Chebyshev rational function.

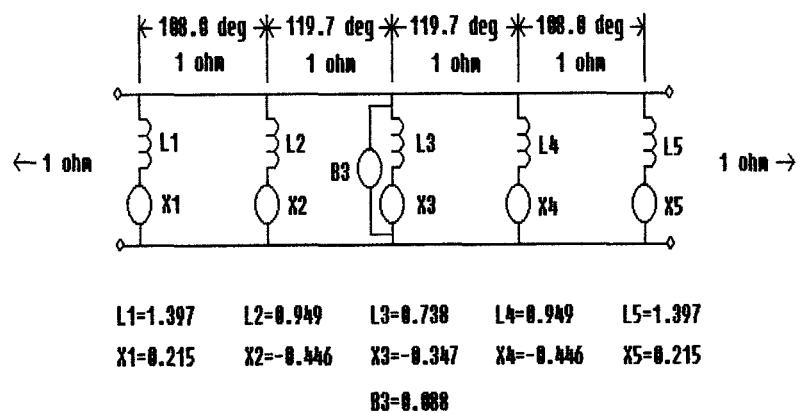


Fig. 3. Normalized highpass prototype.

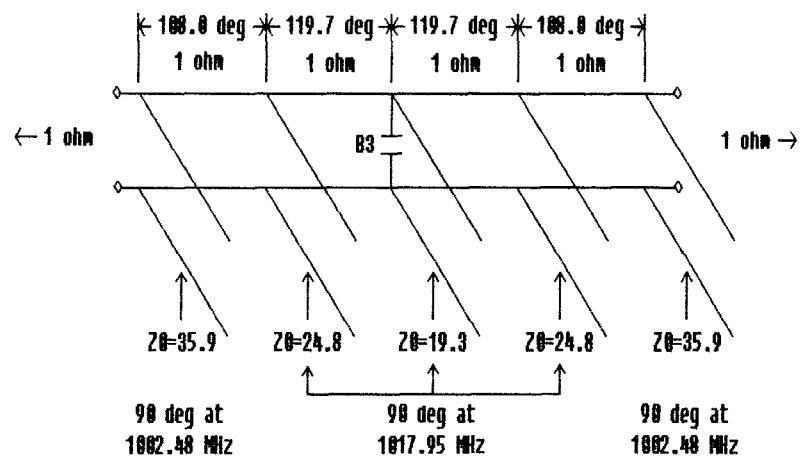


Fig. 4. Normalized transmission line bandstop prototype.

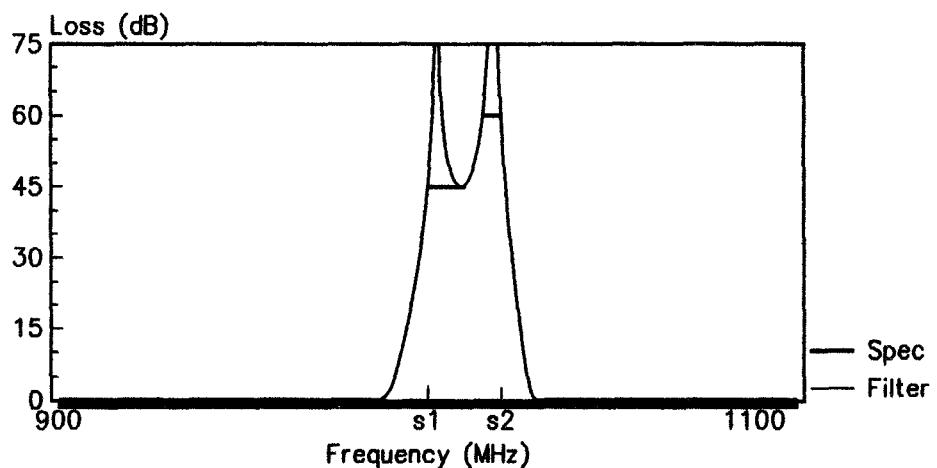


Fig. 5. Loss response, bandstop prototype.

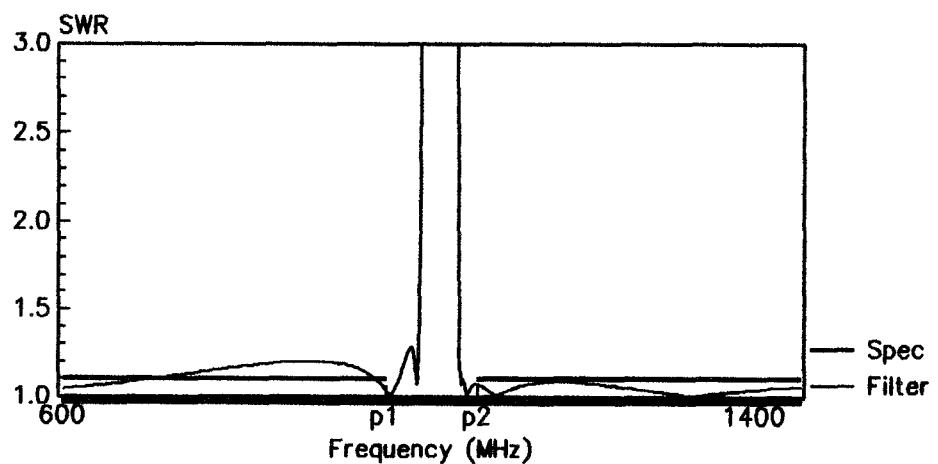


Fig. 6. SWR response, bandstop prototype.